Geomatics Engineering Department

Second Year Geomatics

Geodesy 2 (GED209)

Lecture No: 6

Reduction to Ellipsoid

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Overview Of Previous Lecture

- What is Transformation?
- Datum Transformation
- Transformation Models
- Solution of Equations of Transformation Models
- Datum Shift
- Significance of Datum Transformation
- Take Home Assignment 2

Overview Of Today's Lecture

- Gravimetric Effect
- Geodetic Observations
- Reduction of Azimuth
- Reduction of Horizontal Angles
- Reduction of Vertical Angles
- Reduction of Distances
- Example from Real-World Scenario

EXPECTED LEARNING OUTCOMES

- Gain knowledge about the gravitational field and its influence on geodetic measurements.
- Acquire skills in reducing azimuth observations.
- Learn methods for reducing horizontal angles measured in the field.
- Understand techniques for reducing vertical angles.
- Learn the process of reducing measured distances.

What Do Geodesists Mean By Reduction?

Gravimetric Effect

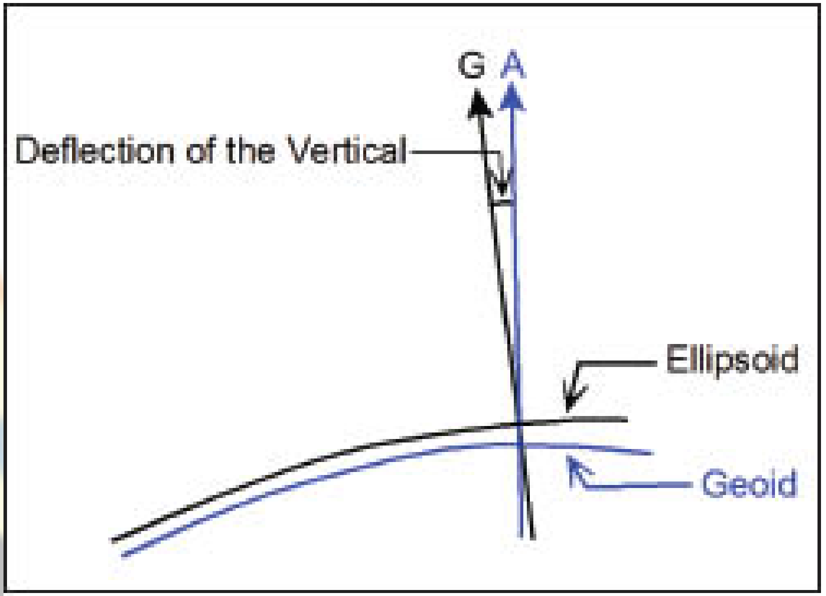
GEODETIC OBSERVATIONS

- When we obtain observations in the field, they are taken with respect to gravity. The gravity helps determine the ***geoid*** but not necessarily the ***ellipsoid***. The ellipsoid is our ***mathematical surface*** for all horizontal control.
- Observations include horizontal angles (e.g., azimuths), zenith distances, spatial distance, and vertical angles.
- GNSS report always include horizontal distances, geodetic distances, and mark-to-mark distances. In terms of azimuths, it also provides geodetic azimuth, grid azimuth, etc.,

What are the differences affect surveying observations?

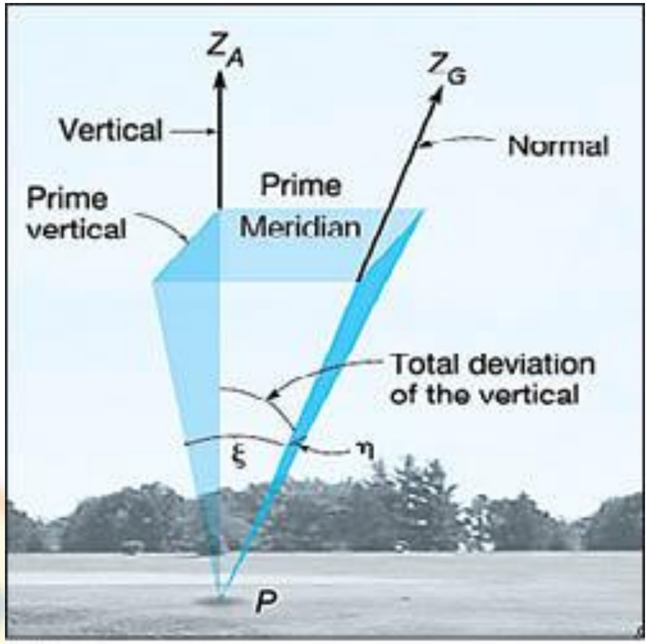
GEODETIC OBSERVATIONS

- Positions determined w.r.t gravity are known as astronomical positions. They are determined from the perpendicular to the geoid (vertical direction), which is marked as A in the shown Figure.
- Positions determined w.r.t the ellipsoid are known as geodetic positions. They are determined w.r.t the normal to the ellipsoid, which is marked as G in the shown Figure.
- Since we level our total station with respect to gravity, we have aligned our instrument with respect to the normal to the geoid (A). These are known as astronomical observations. However, GPS observations and geodetic positions are determined w.r.t the ellipsoid, and therefore G.



GEODETIC OBSERVATIONS

- As shown in Figure, the zenith (ZA) of an astronomical location is determined by the direction opposite the direction of gravity at this location. The zenith (ZG) of a geodetic location is determined by the direction of the normal to the defining ellipsoid. The difference between these two zeniths is known as *deflection of the vertical.*

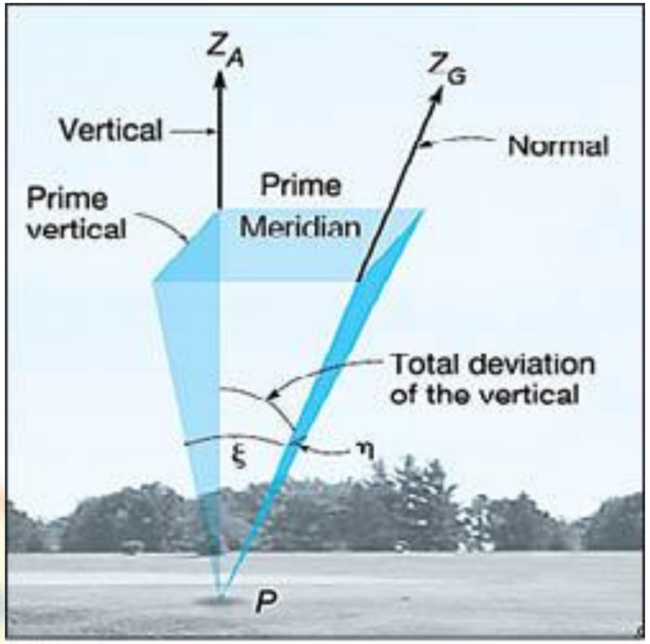


- Typically, the deflection of the vertical is given by two angular components. They are designated by the Greek letters ξ (xi) and η (eta).

- The component of the deflection of the vertical in the north-south direction is ξ and η is the component of the deflection of the vertical in the east-west direction.

GEODETIC OBSERVATIONS

- The ξ component is positive when the astronomical zenith is north of the geodetic zenith, and the η component is positive when the astronomical zenith is east of its geodetic counterpart. This means that if geodetic angles are desired, we need to correct conventional instrument observations, which are oriented with respect to the gravity and the geoid, to the normal of the ellipsoid.



Gravimetric Reduction of Observations

(1) Reduction of Azimuth

(1) Azimuth Reduction

- The astronomic azimuth of the line is affected by the deflection of the vertical at the observing station. It can be reduced to a geodetic azimuth using "Laplace equation": -

$$\Delta\alpha = A - \alpha = \eta \tan\varphi + (\xi \sin\alpha - \eta \cos\alpha)\cot Z \qquad (1)$$

Where:

φ: the geodetic latitude of the observing station.

A: the astronomic azimuth

α: the geodetic azimuth

Z: the observed zenith angle

(1) Azimuth Reduction

- Equation 1 is the well-known "Laplace Equation" in its complete form.
- The first term is the same for every target independent of its azimuth and zenith distance. And it results from the fact that astronomical north rather than from geodetic north as the geodetic azimuth. It thus represents a shift of the zero point, which is the same for all targets.
- Usually in *first-order triangulation* the lines of sight are almost *horizontal*, so that $Z = 90$, $\cot Z = 0$. Therefore, the correction can in general be *neglected* & we thus get: -

$$\Delta\alpha = \eta \tan\varphi \qquad (2)$$

(2) Reduction of Horizontal Angles

(2) Gravimetric Effect on Horizontal Angles

- Any horizontal angle may be considered as the differences between two azimuths. Accordingly, the corresponding horizontal angle W, which refers to the direction of the normal, can be computed as the difference between the two corresponding geodetic azimuths:

$$W_{21} = \alpha_2 - \alpha_1$$

Such that:

$$\alpha_1 = A_1 - \Delta\alpha_1$$

$$\alpha_2 = A_2 - \Delta\alpha_2$$

$$W_{21} = (A_2 - A_1) - (\Delta\alpha_2 - \Delta\alpha_1)$$

$$W_{21} = (A_2 - A_1) - (\xi \sin\alpha_2 - \eta\cos\alpha_2)\cot Z_2 + (\xi \sin\alpha_1 - \eta\cos\alpha_1)\cot Z_1 \qquad (3)$$

- For nearly horizontal lines of sight the difference both zenith angles are very small and can be neglected.

(3) Reduction of Vertical Angles

(3) Gravimetric Effect on Vertical Angles

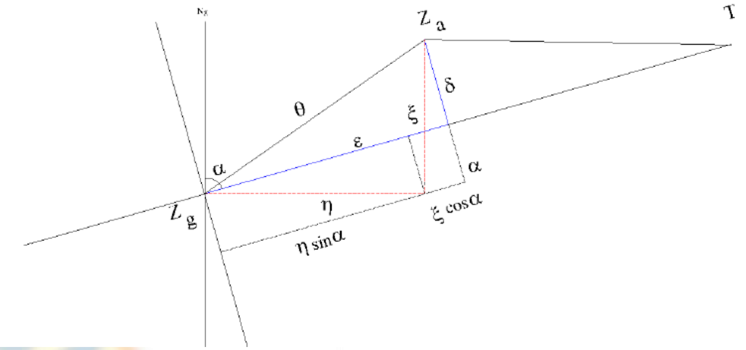
- The deflection of the vertical components in any direction α, ε and δ are related to ξ and η by plane coordinate transformation as follows: -

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha \quad (4)$$

$$\delta = \xi \sin\alpha - \eta \cos\alpha \quad (5)$$

- The relation between geodetic zenith distance $\bar{Z}$ and the “measured” astronomic zenith distance Z is simply given by: -

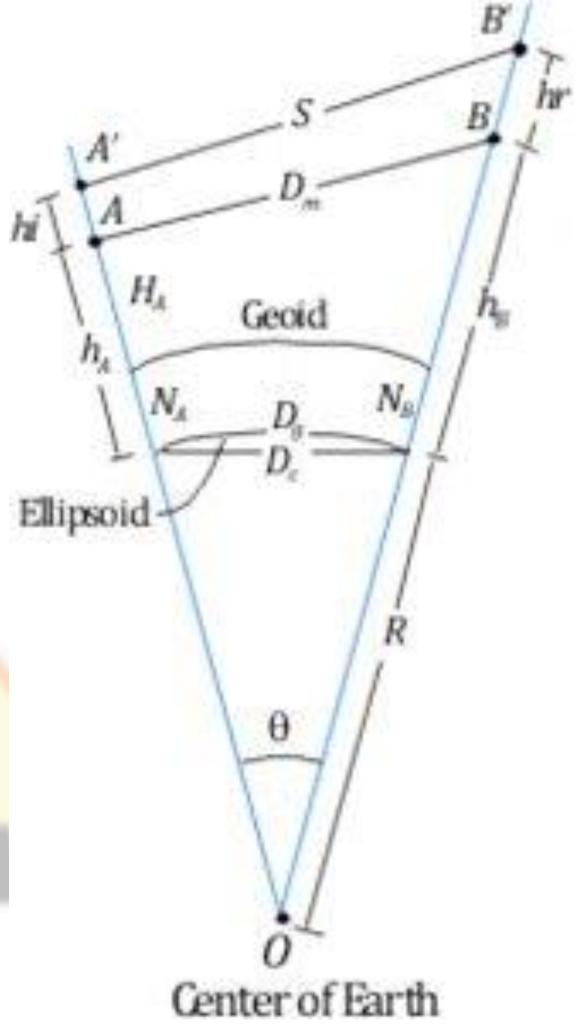
$$Z = \bar{Z} + \varepsilon \quad (6)$$



(4) Reduction of Observed Distances

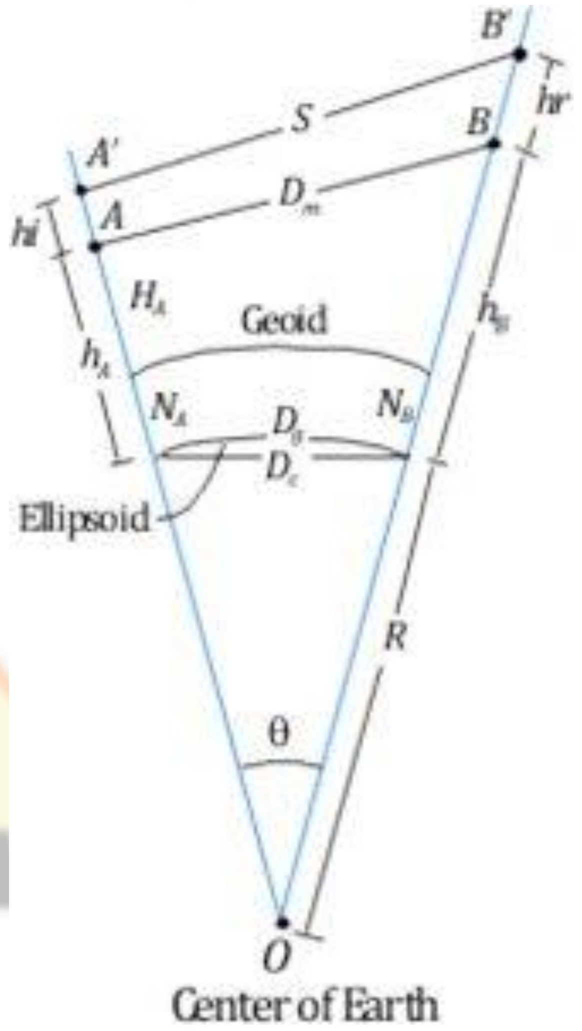
(3) Gravimetric Effect On Observed Distances

- The reduction process is performed in several steps using simple geometric and trigonometric relationships.
- The first reduction takes the observed slope distance and reduces it to a chord length (Dc) at the ellipsoid.
- The second reduction determines the equivalent geodetic distance for the observation. Because the slope distance is observed at some location above the ground and the reflector is similarly off the ground, we need to add their heights to the overall reduction process.



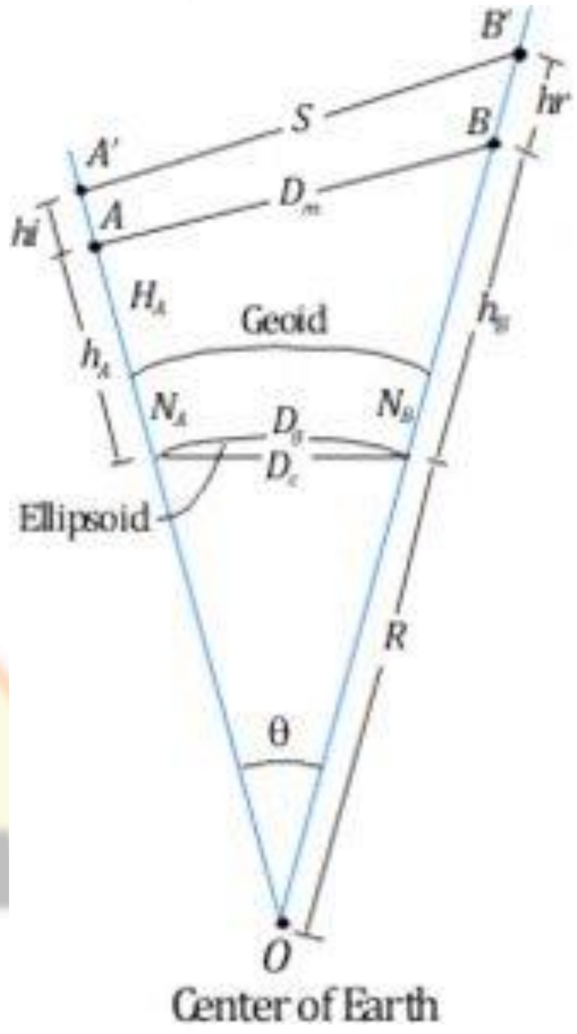
(3) Gravimetric Effect On Observed Distances

- h_A is the geodetic height of the instrument at A, and
- h_B the geodetic height at B;
- hi is the height of the instrument at A, and
- hr the height of the reflector at B;
- N_A the geoidal separation at A, and N_B the geoidal separation at B;
- H_A and H_B are the orthometric heights (commonly referred to as elevations) at A and B, respectively.



(3) Gravimetric Effect On Observed Distances

- The mark-to-mark distance (D_m) and the geodetic distance (D_g) are often reported in GNSS adjustment reports, although not all software manufacturers clearly label these lengths as such.
- Because we need the radius of the Earth R and the geoid separation in this reduction, the example will be worked entirely in metric units.



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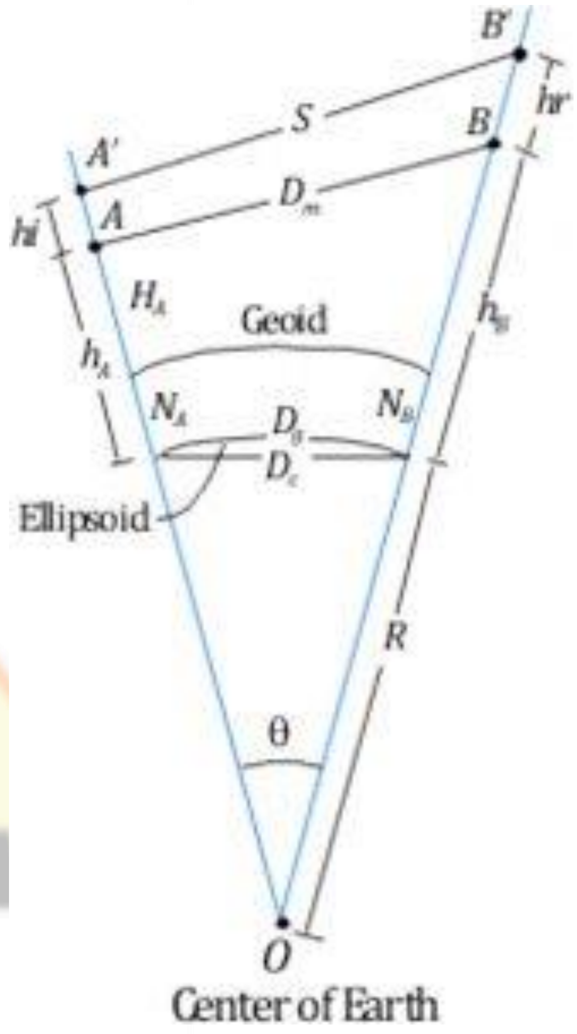
(3) Reduction Of Observed Distances

- Chord Distance

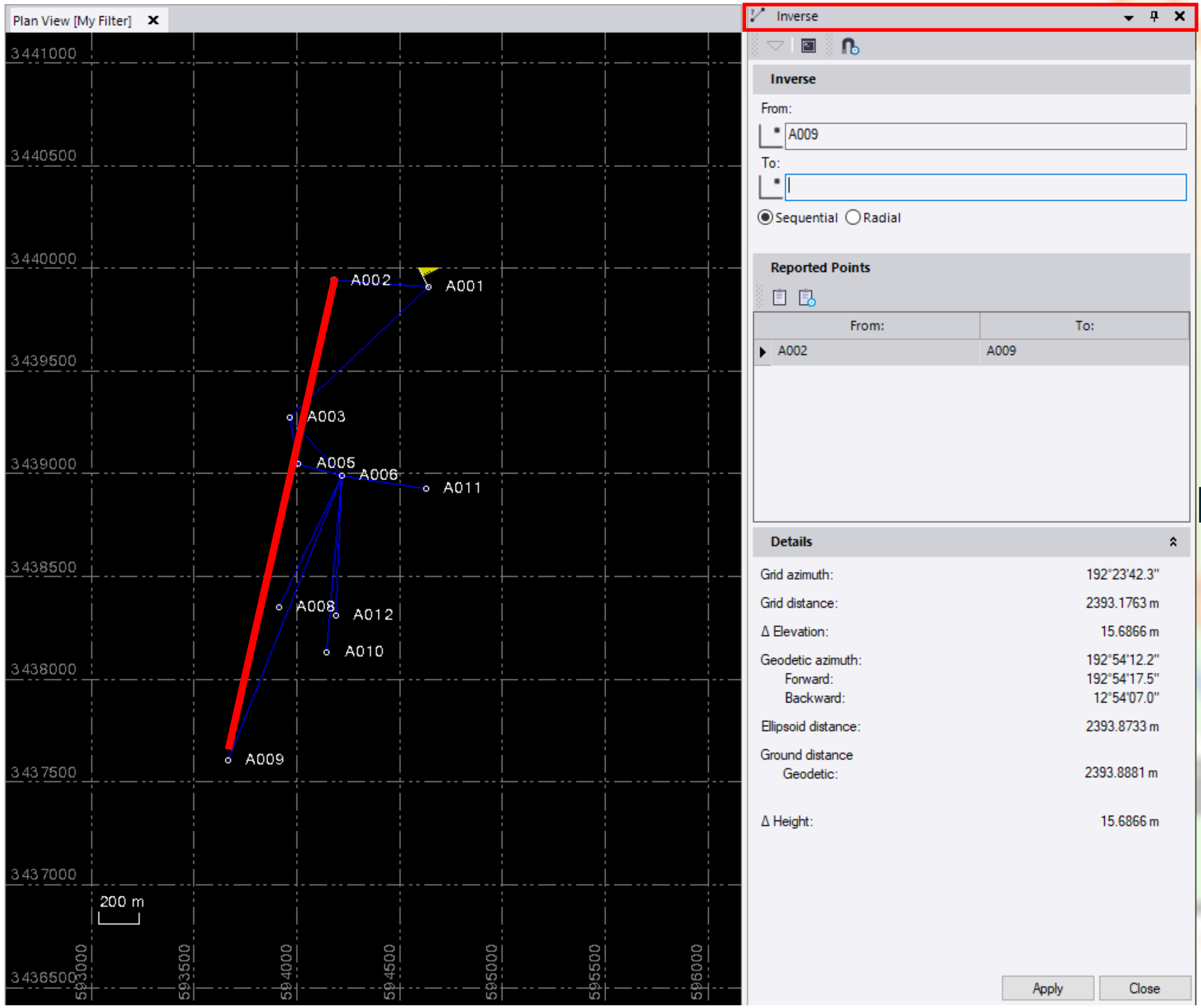
$$D_c = \sqrt{\frac{S^2-(h'_B - h'_A)^2}{(1+\frac{h'_A}{R})(1+\frac{h'_B}{R})}} \qquad (7)$$

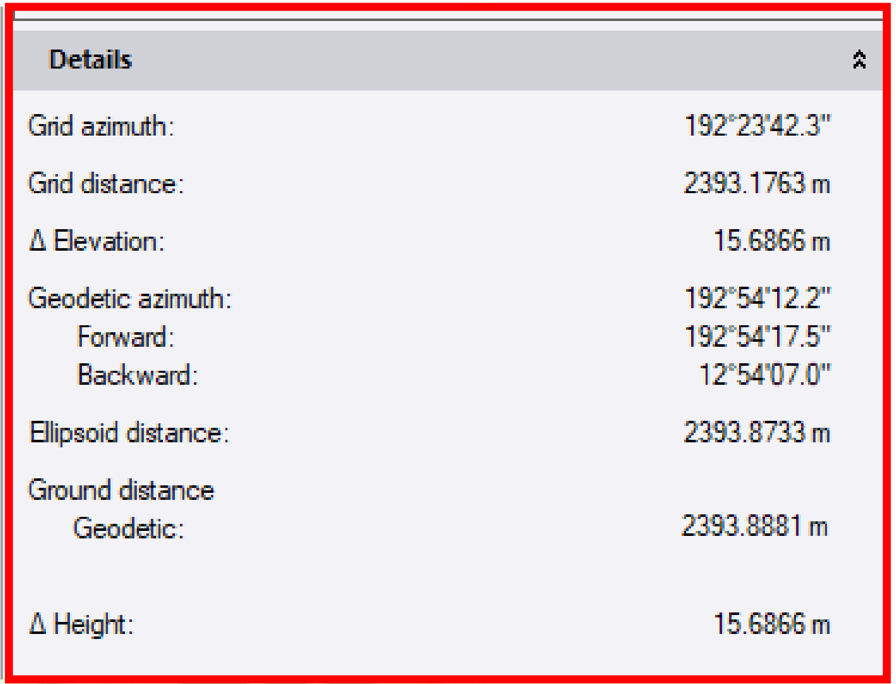


- Geodetic Distance

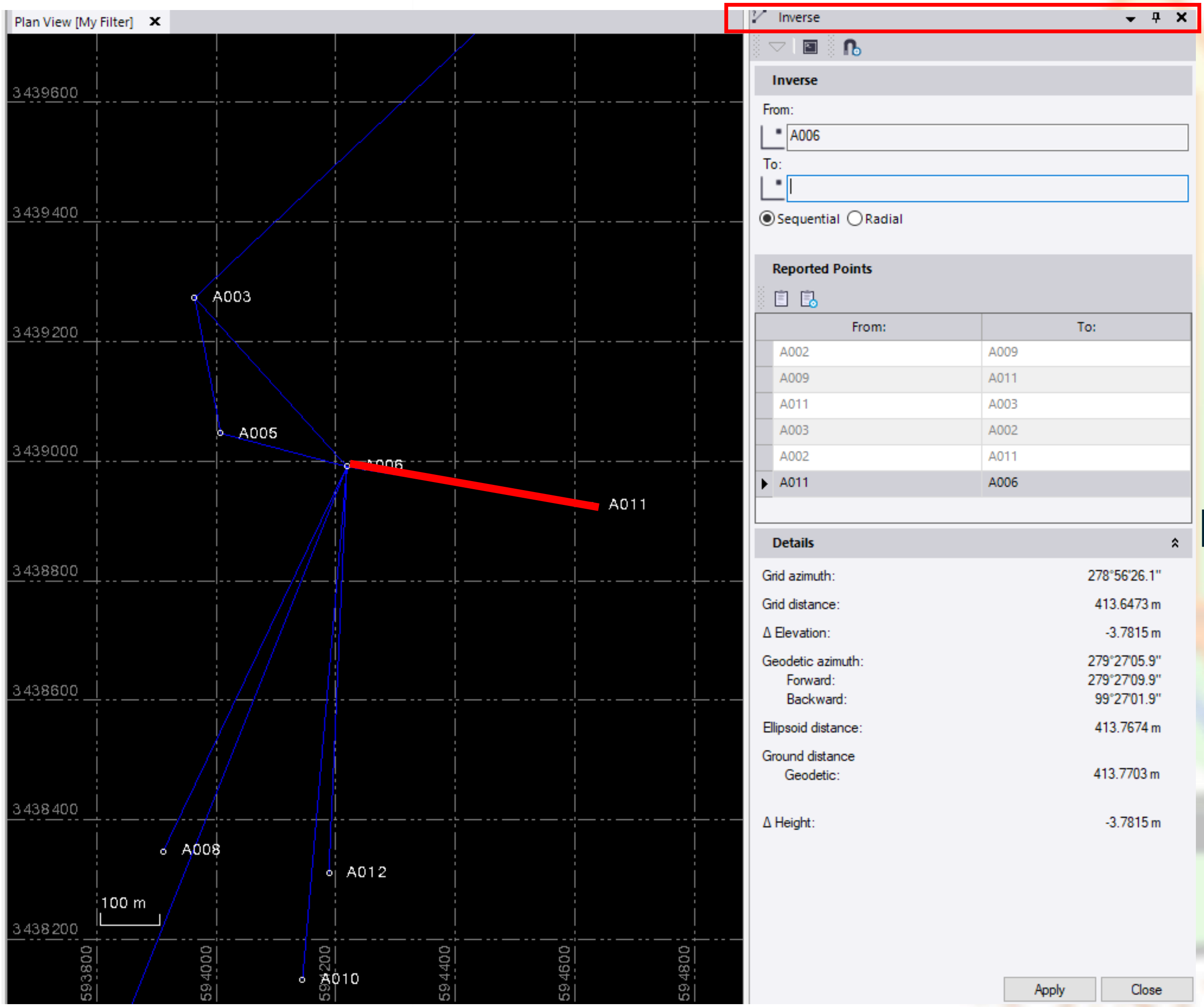
$$D_g = 2R\ \sin^{-1}(\frac{D_c}{2R}) \qquad (8)$$

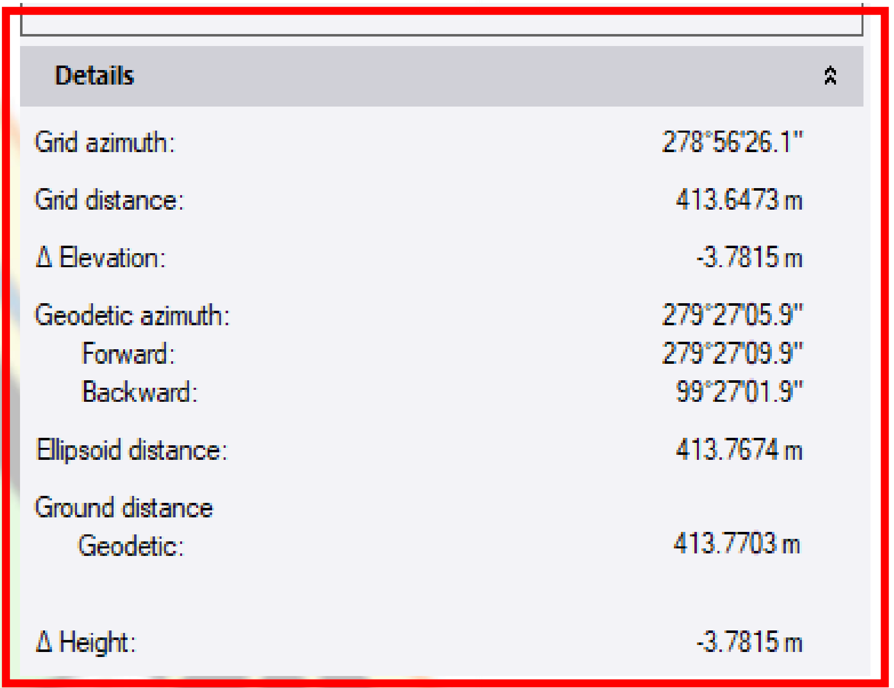
Example from Real-World Scenario





Example from Real-World Scenario





End Of Presentation

Thank You For Attention!